Calculation Methodology

Curves are calculating using standard ‘bootstrapping’ (see below) in conjunction with cublic spline interpolation of the continuously compounded rate. This ensures that the input source rates can be derived exactly from the zero curve and that the instantaneous forward rate is continuous.

Source Instruments
Reuters zero curves are created from the most liquid interest rate instruments that are available: a combination of deposits, liquid futures and interest rate swaps

Deposit rates: ON, TN, SW, 1M, 2M, 3M, 6M, 9M, 1Y
Futures: first 6 International Money Market (IMM) contracts
Interest Rate Swaps: from 2 years to 30 years (where available)

The sources for each currency appear in the chain 0#xxxSZSRCE where xxx is the currency code.

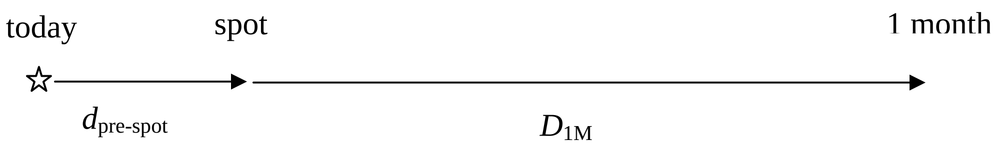
Bootstrapping

Any combination of additive cash flows can be shown to be a combination of zero coupon cash flows. Splitting an instrument into its constituent zero coupon cash flows is important because an instrument with multiple cash flows should not be discounted using the same interest rate from settlement date to maturity. This ‘stripping’ of the curve and the successive calculation of discount factors (using the equations in the next section) is termed bootstrapping.

Intermediate points on the curve can be estimated by assuming a shape for the curve either in discount factor or rate space. Linear, exponential and cubic splining are different interpolation methods that will give different curve shapes between known points. Reuters uses cubic spline interpolation of discount factors.

Discount factors from deposit rates

All Reuter swap-based spot curves start from today. However, the underlying cash rates start from the money market spot date which, depending on the currency, may be two business days later[1]. A small correction needs to be made for discounting this period of time. This pre-spot discount factor ‘stub’ can be calculated from the ON, TN rates.



Discount factors are related to money market deposit rates by the following simple interest formula:

[1] Adopting the same start date for all Reuters spot curves allows for simpler cross-country comparisons. For currencies with a money market spot of ‘today’, for example Sterling, this correction does not apply.

$$d = \frac{1}{1 + (r \times {}^{n}\!/\!_{Y})} \qquad (1)$$

where : *d* is the discount factor for the period
r is the money market annual rate of interest
n is the actual number of days in the period
Y is the year basis (e.g. 365 for THB)

The 1-month discount factor d_{1MZ} is calculated as the product

$$d_{1MZ} = d_{prespot} \times d_{1M} \qquad (2)$$

Deposit rates for tenors SW, 1M, 2M …. are calculated in this way. Then the discount factor is represented as a zero rate on a compounded actual/actual (period-based) basis using:

$$r = \left(\frac{1}{d}\right)^{1/f} - 1 \qquad (3)$$

where : *d* is the discount factor for the period
r is the compounded annual rate of interest
f is the number of years (including fraction of year)

Discount factors from futures

For currencies with liquid short term interest rate (STIR) futures, these prices are used as sources to the zero curve between 1 – 3 months and approximately 2 years (depending on liquidity).

Liquid International Money Market (IMM) STIR futures contracts are used which have contract expiry dates in March (H), June (M), September (U) and December (Z).

The futures price from a STIR contract gives us the forward rate from the contract expiry/settle date to the end of the forward rate period, for example, a March 3M Euribor futures contract expires in March and the underlying forward rate end date is 3 months later. The discount factor for this period can be calculated from equation (1) such that

$$d_{future} = \frac{1}{1 + \frac{(100 - price)n}{Y}} \tag{4}$$

If in this example, our first IMM futures contract expired in 2.5 months time, it would be necessary to calculate the discount factor up to the expiry date of the contract. This can be estimated by interpolating the discount factors of the 2M and 3M deposit rates, d_{i23} (not forgetting the pre-spot stub). It is then appropriate to calculate the discount factor, d_{fz}, from zero curve start date to end date of the forward rate as follows :

$$d_{fz} = d_{i23} \times d_{future} \tag{5}$$

and so on until the end of the futures strip.If the end and start date of the underlying forward rate of consecutive futures overlap then the discount factor of the start date of the next future must be interpolated from these discount factors already calculated. Alternatively, if there is a gap between end and start dates of consecutive futures then the discount factor at the start date of the next future must be interpolated using discount factors derived from deposit rates. Adfin does this interpolation automatically if the *AdTermStructure* function is used with appropraite inputs – see below.

Discount factors from Interest Rate Swaps

The present value (*V*) of a fixed for floating interest rate swap is given by the following formula :

$$V = \sum_{j=1}^{M} d(t_j).Q(t_j) - \sum_{i=1}^{N} d(t_i).P(t_i) \tag{6}$$

where: $Q(t_j)$ are the M fixed rate payments
$P(t_i)$ are the N floating rate payments
$d(t_j)$ are the discount factors for dates t_j
$d(t_i)$ are the discount factors for dates t_i

At the initial outset, swaps trade at par so *V = 0*. The array of rates and discount factors has already been calculated up to the end date of the underlying of the last futures contract. Therefore, an iterative technique can then be used to solve for the next unknown discount factor – in this case the 2Y discount factor – the value of which forces the net present value of the swap to 0.

Interpolation
There are market swap quotes for annual tenors but in this example the interest rate swap produces cashflows every 6 months according to the payment frequency. This means that the 6-monthly fixed rate for non-annual points must be estimated and used as input into the above cashflow calculation. The values are estimated by cubic-spline interpolation between the known fixed rates.

Adfin Term Structure Calculations

Adfin analytics provide tools to calculate the term structure of interest rates. The function *AdTermStructure* can be employed to generate an array of dates and rates/discount factors. Online help in Power Plus Pro describes this funtion in detail. Alternatively, the *Zero Coupon Builder* model facilitates the easy construction of zero coupon curves.

The function takes an array of source data, *InputArray*, as follows:

Type	StartDate	Maturity	Coupon	Market Price and Rate	Structure
"D" for deposits	Start date of the period	End date of the period	N/A	Rate	Currency Code
"F" for futures	Start date of the period	End date of the period	N/A	Implied forward rate	Adfin STIR Future style
"S" for IRS	Start date of the swap	Maturity code or maturity date	Current floating rate or zero	Fixed swap rate	Adfin Swap Structure

Calculating Inputs: start, maturity dates and market rates

Start and end dates can be calculated using Adfin functions as follows:

For deposits
Use of *FxCalcPeriod* – an array is returned with RET:a2

start, maturity = *FxCalcPeriod(Today,Curr, Period, "FROM:MMTRADE RET:a2")*

returning a 2D array. The input rate is the percentage rate divided by 100, or expressed as a % in Excel..

For futures
Use of function *AdFutDates(Adfin Futures Style, Future Contract Code)* returns the start and end dates of the period of the implied forward rate. The implied forward rate is given by *(100 – price)/100*

For swaps
Start date is given by the spot date calculated from *FxCalcPeriod* above and the maturity given by the maturity code, 2Y, 3Y and so on. The market rate is the quoted fixed rate/100.

An example *InputArray* is shown here:

Instrument Type	Start Date	Maturity	First rate	Market rate	Instrument Structure
D	26 Mar 2004	29 Mar 2004		2.19%	EUR
D	29 Mar 2004	30 Mar 2004		2.04%	EUR
D	30 Mar 2004	06 Apr 2004		2.09%	EUR
D	30 Mar 2004	30 Apr 2004		2.05%	EUR
D	30 Mar 2004	31 May 2004		1.99%	EUR
D	30 Mar 2004	30 Jun 2004		1.97%	EUR
D	30 Mar 2004	30 Sep 2004		1.97%	EUR
D	30 Mar 2004	30 Dec 2004		1.95%	EUR
F	16 Jun 2004	16 Sep 2004		1.84000%	FEI
F	15 Sep 2004	15 Dec 2004		1.85000%	FEI
F	15 Dec 2004	15 Mar 2005		1.95000%	FEI
F	16 Mar 2005	16 Jun 2005		2.10500%	FEI
F	15 Jun 2005	15 Sep 2005		2.31500%	FEI
F	21 Sep 2005	21 Dec 2005		2.52500%	FEI
S	30 Mar 2004	2Y	0.0000%	2.2200%	EUR_AB6E
S	30 Mar 2004	3Y	0.0000%	2.5450%	EUR_AB6E
S	30 Mar 2004	4Y	0.0000%	2.8500%	EUR_AB6E
S	30 Mar 2004	5Y	0.0000%	3.1250%	EUR_AB6E
S	30 Mar 2004	6Y	0.0000%	3.3500%	EUR_AB6E
S	30 Mar 2004	7Y	0.0000%	3.5550%	EUR_AB6E
S	30 Mar 2004	8Y	0.0000%	3.7300%	EUR_AB6E
S	30 Mar 2004	9Y	0.0000%	3.8750%	EUR_AB6E
S	30 Mar 2004	10Y	0.0000%	3.9950%	EUR_AB6E
S	30 Mar 2004	11Y	0.0000%	4.0999%	EUR_AB6E
S	30 Mar 2004	12Y	0.0000%	4.1935%	EUR_AB6E
S	30 Mar 2004	13Y	0.0000%	4.2742%	EUR_AB6E
S	30 Mar 2004	14Y	0.0000%	4.3456%	EUR_AB6E
S	30 Mar 2004	15Y	0.0000%	4.4085%	EUR_AB6E
S	30 Mar 2004	16Y	0.0000%	4.4649%	EUR_AB6E
S	30 Mar 2004	17Y	0.0000%	4.5159%	EUR_AB6E
S	30 Mar 2004	18Y	0.0000%	4.5616%	EUR_AB6E
S	30 Mar 2004	19Y	0.0000%	4.6022%	EUR_AB6E
S	30 Mar 2004	20Y	0.0000%	4.6375%	EUR_AB6E
S	30 Mar 2004	21Y	0.0000%	4.6669%	EUR_AB6E
S	30 Mar 2004	22Y	0.0000%	4.6897%	EUR_AB6E
S	30 Mar 2004	23Y	0.0000%	4.7081%	EUR_AB6E
S	30 Mar 2004	24Y	0.0000%	4.7235%	EUR_AB6E
S	30 Mar 2004	25Y	0.0000%	4.7375%	EUR_AB6E
S	30 Mar 2004	30Y	0.0000%	4.7775%	EUR_AB6E

The output array (dates and rates) is generated by calling the function as follows:

AdTermStructure (InputArray, Ratestructure, Array size)

Where the ratestructure is given by:

RM:YC ZCTYPE:DF IM:CUBD ND:DIS

which defines that discount factors will be returned and cubic spline interpolation should be used.